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उद्धव बाळासाहेब ठाकरे

ANSWER KEY & SOLUTIONS
TO
MHT – CET MOCK

(Physics, Chemistry & Mathematics)

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MHT - CET MOCK TEST - 2025

Subjects : Physics, Chemistry & Mathematics

1. Answer key is provided to all the questions.
2. Solutions are provided below the Answer Key, wherever needed.

1. (B)

$$B = \frac{\mu_0 I}{2\pi r}$$

New distance $= \frac{r}{2}$

∴ New magnetic field $= \frac{\mu_0}{4\pi}\frac{2I}{\left(\frac{r}{2}\right)} = 2B$

2. (C)

In an unmagnetized ferromagnetic substance, the magnetic domains are randomly oriented in all directions. This means that the magnetic moments of the atoms within each domain point in different directions, resulting in a net magnetic moment of zero. This random orientation is the reason why the material is not magnetically polarized. If the domains were aligned, they would create a net magnetic moment and the substance would become magnetic.

3. (C)

$$\tan\phi = \frac{X_L - X_C}{R} = \frac{500-200}{100} = \frac{300}{300}$$

∴ $\tan\phi = 1 \Rightarrow \phi = 45°$

4. (B)

For photon,

$$E = \frac{hc}{\lambda}$$

∴ For electron,

$$\lambda' = \frac{h}{\sqrt{2mE}} = \frac{h}{\sqrt{2m\frac{hc}{\lambda}}} = \sqrt{\frac{h\lambda}{2mc}}$$

5. (A)

In a CE amplifier, the input signal is applied to the base of the transistor, which is the emitter-base junction. The base-emitter junction is forward biased for the amplifier to operate in the active region. The amplified output signal is then taken from the collector, which is connected to the collector-base junction. Therefore, the input AC signal in a CE amplifier is applied across the forward-biased emitter-base junction.

6. (D)

The behaviour of an ideal blackbody is similar to a group of electromagnetic oscillators that emit energy in quanta, hence quantum oscillators. This is because Planck proposed that the atoms on the walls of the cavity act as tiny electromagnetic oscillators with different characteristic frequencies, exchanging energy with the cavity. The energy is given by $E = nh\nu$, where ν is the frequency, h is a universal constant $\left(6.626\times10^{-34}\,\text{J s}\right)$, and is a positive integer. These oscillators do not radiate energy continuously but only in "jumps" or "quanta" corresponding to transitions from one quantized energy level to another.

7. (C)

$\tan\theta_B = n = 1.55$

∴ $\theta_B = 57°10'$

$r = 90° - \theta_B = 90° - 57°17' = 32°49'$

8. (A)
When gas molecules collide with the walls of a container, they exert a force on the walls and, in turn, experience a force from the walls. This exchange of momentum between the molecules and the walls is the basis of gas pressure. The molecules do not lose their momentum completely, but rather transfer a portion of it to the walls.

9. (C)
$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$.
mu + 3m × 0 = m × 0 + 3m × v_2

∴ $v_2 = \frac{u}{3}$

$$e = \frac{v_2 - v_1}{u_1 - u_2} = \frac{\frac{u}{3} - 0}{u - 0} = \frac{u/3}{u} = \frac{1}{3}$$

10. (A)
At the extreme position, the displacement of the oscillator is maximum. Since potential energy depends on displacement, it is also maximum at this point. As the total energy of the system remains constant, if potential energy is maximum, the kinetic energy must be zero (minimum) at the extreme position to maintain the total energy balance.

11. (A)
Limit of resolution $\theta = \frac{1.22\lambda}{D} = \frac{1.22 \times 4000 \times 10^{-10}}{0.05} = 9.76 \times 10^{-6}$ rad

12. (A)
The time of ascent is equal to the time of descent because the acceleration due to gravity remains constant throughout the motion, and the initial velocity at the start of the ascent is the same as the final velocity at the end of the descent. The only force acting on the object is gravity. Therefore, the time taken for the object to go up is the same as the time taken for it to come down. $t_a = t_d$

13. (C)
Path difference for all wavelengths at the central point will be zero. Hence, central band will be white a few coloured bands are observed on either side of the central band.

14. (B)
$C = \frac{Q}{V}$

but $V = \frac{1}{4\pi\varepsilon_0}\frac{Q}{R}$ for a spherical body.

∴ $C = 4\pi\varepsilon_0 R$
∴ $C = 4\pi \times 8.85 \times 10^{-12} \times 6400 \times 10^3$
∴ $C = 7.1 \times 10^{-4}$ F

15. (C)
$$E = \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{4^2} + ...\right]$$
Using Binomial theorem,
$$E = \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{1 - \frac{1}{4}}\right] = 5 \times 9 \times 10^9 \times \frac{4}{3} = 6 \times 10^{10} \text{ N/C}$$

16. (B)

Velocity of efflux when the hole is at depth h,

$v = \sqrt{2gh}$

Rate of flow of water from square hole

$Q_1 = a_1 v_1 = L^2\sqrt{2gy}$

Rate of flow of water from circular hole

$Q_2 = a_2 v_2 = \pi R^2\sqrt{2g(4y)}$

According to problem $Q_1 = Q_2$

$\Rightarrow L^2\sqrt{2gy} = \pi R^2\sqrt{2g(4y)} \quad \Rightarrow R = \frac{L}{\sqrt{2\pi}}$

17. (A)

$\lambda = \frac{h}{mv}$

∴ $\lambda \ \alpha \ \frac{1}{v}$

Now, $v \ \alpha \ \sqrt{T}$

∴ $\lambda \ \alpha \ \frac{1}{\sqrt{T}}$

$\Rightarrow \frac{\lambda_{27}}{\lambda_{927}} = \sqrt{\frac{T_{927}}{T_{27}}}$

$\Rightarrow \lambda_{27} = 2\,\lambda_{927}$

$\Rightarrow \lambda_{927} = \frac{\lambda_{27}}{2} = \frac{\lambda}{2}$

18. (C)

$\frac{2}{5}MR_s^2 = \frac{2}{3}MR_h^2$

∴ $\frac{R_s}{R_h} = \frac{\sqrt{5}}{\sqrt{3}}$

19. (B)

Magnetic field at the centre of circular coil, ($L = 2\pi r_1$)

$B_{circular} = \frac{\mu_0}{4\pi}\frac{2\pi i}{r_1} = \frac{\mu_0}{4\pi}\frac{4\pi^2 i}{L}$ and

Magnetic field at the centre of semi-circular coil, ($L = \pi r_2$)

$B_{semi\text{-}circular} = \frac{\mu_0}{4\pi}\frac{\pi i}{r_2} = \frac{\mu_0}{4\pi}\frac{\pi^2 i}{L}$

∴ $\frac{B_{circular}}{B_{semi\text{-}circular}} = 4$

20. (B)

$C_{net} = 5\ \mu F$

$Q_{net} = 5 \times 8 = 40\ \mu C$

We know,

$Q_{2\mu F} = 2 \times 8 = 16\mu C$

∴ $Q_{4\mu F} = Q_{12\mu F} = Q_{net} - Q_{2\mu F}$ ...{∵ $9\mu F \| 3\mu F = 12\mu F$}

$= 40 - 16 = 24\mu C$

Voltage across 4μF and 12μF can be given as,

$V_{4\mu F} + V_{12\mu F} = V$

$\therefore \quad V_{4\mu F} = V - \frac{Q_{12\mu F}}{C_{12\mu F}} = 8 - \frac{24}{12} = 6V$

$\therefore \quad V_{12}\mu F = 2V$

i.e. $V_{9\mu F} = 2V$

$\therefore \quad Q_{9\mu F} = 9 \times 2 = 18\mu C$

$\therefore \quad Q = Q_{4\ \mu F} + Q_{9\ \mu F} = 42\mu C$

$\therefore \quad E = \frac{kQ}{r^2} = \frac{9 \times 10^9 \times 42 \times 10^{-6}}{30\times30} = 420\ N/C$

21. (D)

$\chi = (\mu_r - 1)$

$\therefore \quad \chi = (400 - 1) = 399$

22. (D)

Self inductance of a solenoid is given by

$L = \mu_0 n^2 l A = \frac{\mu_0 N^2 A}{l} = \frac{\mu_0 N^2 \pi r^2}{l} \qquad \ldots\left(\because n = \frac{N}{l}\right)$

N is same for both.

$\therefore \quad \frac{L_1}{L_2} = \left(\frac{r_1}{r_2}\right)^2 \left(\frac{l_2}{l_1}\right) = \left(\frac{1}{2}\right)^2 (2) = \frac{1}{2}$

23. (B)

$$n = \frac{\sin\left(\frac{A+\delta_m}{2}\right)}{\sin\left(\frac{A}{2}\right)}$$

$$\sin\left(\frac{A+\delta_m}{2}\right) = n \sin\left(\frac{A}{2}\right)$$

$$\sin\left(\frac{60^o + \delta_m}{2}\right) = 1.4 \sin\left(\frac{60^o}{2}\right)$$

$$\sin\left(\frac{60^o + \delta_m}{2}\right) = 0.6$$

$$30° < \frac{60^o + \delta_m}{2} < 45°$$

$$60° < 60° + \delta_m < 90°$$

$$0° < \delta_m < 30°$$

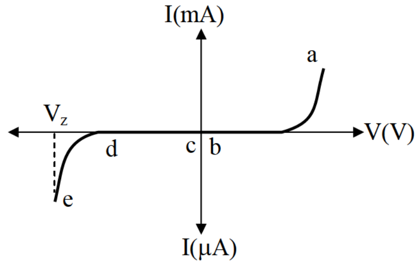
24. (C)

$Z = \sqrt{R^2 + X_L^2} = 10\sqrt{2}\ \Omega$

$e_0 = \sqrt{2}e = 220\sqrt{2}\ V$

$\therefore \quad i_0 = \frac{e_0}{Z} = \frac{220\sqrt{2}}{10\sqrt{2}} = 22\ A$

25. (D)



When the reverse bias is greater than the V_z, it is breakdown condition. In breakdown region, ($V_i > V_z$) for a wide range of load; (R_L), the voltage across R_L remains constant though the current may change. Hence portion 'de' of the characteristics is relevant for the zener diode to operate as a voltage regulator.

26. (C)

We know for radioactive decay,

$N = N_0e^{-\lambda t}$ (or) $ln \dfrac{N_0}{N} = \lambda t$

For 20% decay

$t = \dfrac{1}{\lambda}\ ln \dfrac{N_0}{N}$

$\Rightarrow t = \dfrac{20}{0.693}\left(ln \dfrac{100}{20}\right) \qquad \ldots\left(\because \lambda = \dfrac{0.693}{T}\right)$

$\Rightarrow t = \dfrac{20}{0.693}\ ln\ (5) \qquad \ldots(i)$

For 80% decay

$t' = \dfrac{1}{\lambda}\ ln \dfrac{N_0}{N'}$

$\Rightarrow t' = \dfrac{20}{0.693}\ ln\left(\dfrac{100}{80}\right)$

$\Rightarrow t' = \dfrac{20}{0.693}\ ln\left(\dfrac{5}{4}\right) \qquad \ldots(ii)$

Thus, $\Delta t = t - t'$

$= \dfrac{20}{0.693}\left(ln 5 - ln\dfrac{5}{4}\right)$

$= \dfrac{20}{0.693}\ ln\ 4 = 40$ min.

27. (B)

$R_m = \dfrac{R_e}{2},\ \rho_m = \dfrac{1}{4}\rho_e$

Energy spent $= mg_eh_e = mg_m\ h_m$

∴ $h_m = g_eh_e / g_m$

∴ $h_m = \dfrac{\left(\dfrac{4}{3}\pi R_e\rho_e G\right)\times h_e}{\dfrac{4}{3}\pi R_m\rho_m G}$

∴ $h_m = \dfrac{R_e}{R_m} \times \dfrac{\rho_e}{\rho_m} \times h_e = \dfrac{2}{1} \times \dfrac{4}{1} \times 0.5$

$= 4$ m

28. (D)

$$\frac{P_1}{\rho g}+\frac{v_1^2}{2g}=\frac{P_2}{\rho g}+\frac{v_2^2}{2g}$$

$$\frac{4.5\times10^5}{\rho g}+0=\frac{4\times10^5}{\rho g}+\frac{1v^2}{2g}$$

$$v_2^2=\frac{10^5}{\rho}=\frac{10^5}{10^3}$$

$v_2 = 10$ m s^{-1}

29. (D)

According to Kepler's law $T^2 \propto R^3$

If n is the frequency of revolution then

$n^2 \propto (R)^{-3}$

$\therefore \quad \frac{n_2}{n_1}=\left(\frac{R_2}{R_1}\right)^{-3/2} \Rightarrow \frac{R_1}{R_2}=\left(\frac{n_2}{n_1}\right)^{2/3}$

30. (D)

Wavelength will be minimum for $n = \infty$ and energy emitted will be maximum for this transition.

31. (A)

$n'_{driver} = n_{horn}\left(\frac{v+v_C}{v-v_C}\right)$ where v_C : velocity of car

$= 500\left(\frac{330+30}{330-30}\right)$

$= 600$ Hz

32. (A)

Applying Kirchhoff law,

$(2 + 2) = (0.1 + 0.3 + 0.2)I$

$\therefore \quad I = \frac{20}{3}$ A

$\therefore$ Potential difference across A

$= 2 - 0.1 \times \frac{20}{3} = \frac{4}{3}$ V (less than 2 V)

Potential difference across B

$= 2 - 0.3 \times \frac{20}{3} = 0$

33. (B)

$P_P = P_S = I_SE_S$

$\therefore \quad I_S = \ = 5$ A

Now, $\frac{N_S}{N_P}=\frac{I_P}{I_S}$

$\therefore \quad N_S = \frac{N_PI_P}{I_S}$

$= \frac{1000\times0.1}{5}$

$= 20$

34. (B)

$$\eta_{max} = 1 - \frac{T_C}{T_H}$$

$$= 1 - \frac{300}{400} = \frac{1}{4} = 25\%$$

$\Rightarrow$ 24% efficiency is possible.

35. (A)

$$E = \frac{1}{2} m\omega^2 x^2 = \frac{1}{2} m \frac{4\pi^2}{T^2} x^2$$

$$\therefore \quad T = \sqrt{\frac{2m}{E}} \pi x$$

$$= \sqrt{\frac{2 \times 0.2}{4 \times 10^{-5}}} \times 2 \times 10^{-2} \times \pi$$

$$= 100 \times 2 \times 10^{-2}\pi$$

$$= 2\pi \text{ seconds}$$

36. (A)

Phase difference = $\frac{2\pi}{\lambda}$ × Path difference

$$\therefore \quad \frac{\pi}{4} = \frac{2\pi}{\lambda} \times x$$

$$\therefore \quad \frac{\lambda}{4} = x$$

Given equation is,

y = 0.04 sin (600πt + 1.5πx)

Comparing it with standard wave equation,

$$y = A \sin\left(\frac{2\pi t}{T} + \frac{2\pi x}{\lambda}\right)$$

$$\frac{2\pi}{\lambda} = 1.5\,\pi$$

$$\therefore \quad \frac{\lambda}{4} = \frac{1}{1.5} = 2.66$$

$$\therefore \quad x = 2.66 \text{ m}$$

37. (B)

M.I. of thin Rod about one end, $I = \frac{ML^2}{3}$

Now, $L = 2\pi R \Rightarrow R = \frac{L}{2\pi}$

M.I. of ring about diameter,

$$I_1 = \frac{MR^2}{2} = \frac{M\left(\frac{L^2}{4\pi^2}\right)}{2} = \frac{ML^2}{8\pi^2}$$

$$\therefore \quad \frac{I}{I_1} = \frac{ML^2}{3} \times \frac{8\pi^2}{ML^2} = \frac{8\pi^2}{3}$$

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38. (B)

For the diffraction minima, $a \sin\theta = n\lambda$

For 1[st] minima $n = 1$

∴ $a \sin\theta = \lambda$

∴ $a = \frac{\lambda}{\sin\theta} \quad \frac{4.2 \times 10^{-7}}{\sin 30^{\circ}} = 2.1 \times 10^{-7}$ m

∴ $a = 0.21 \times 10^{-6}$ m = 0.21 micron

39. (B)

Induced e.m.f., $e = n\frac{d}{dt}(BA)$

For A = constant, induced e.m.f. is,

$e = nA\frac{dB}{dt} = 50 \times (0.1)^2 \times 1 = 0.5$ V

40. (B)

$P = E_{rms} I_{rms} \cos\phi$

$\cos\phi = \frac{R}{Z}$

Also, $I_{rms} = \frac{E_{rms}}{Z} = \frac{E_0}{Z\sqrt{2}}$

∴ $P = \frac{E_0}{\sqrt{2}} \times \frac{E_0}{Z\sqrt{2}} \times \frac{R}{Z} = \frac{E_0^2 R}{2Z^2}$

Given $X_L = R$

∴ $Z = \sqrt{R^2 + R^2} = \sqrt{2}R$

∴ $P = \frac{E_0^2 R}{2(2R^2)} = \frac{E_0^2}{4R}$

41. (D)

∴ There are two NOR gates, one NOT gate, and one NAND gate.

∴ Output of NAND gate: $\overline{A \cdot B}$

Output of NOT and NOR Gate: $\overline{\overline{A} + B}$

Final output: $\overline{\left(\overline{A \cdot B}\right) + \left(\overline{\overline{A} + B}\right)}$

So, the output Y is 1 only if the input A and B is 1.

$A = 1$

$B = 1$

$Y = \overline{\overline{1 \cdot 1} + \overline{1} + 1}$

$Y = 1$

42. (B)

Height to which water rises in a capillary tube is given by $h = \frac{2T}{r\rho g} = \frac{4T}{d\rho g}$ … ($\because d = 2r$)

∴ $h_1 - h_2 = \frac{4T}{\rho g}\left[\frac{1}{d_1} - \frac{1}{d_2}\right] = \frac{4T}{\rho g}\left[\frac{d_2 - d_1}{d_1 d_2}\right]$

43. (C)

From lens maker's equation, we can write

$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

Given n = 1.5, $R_1' = R_2' = 5\text{cm}$

and $R_1'' = R_2'' = 6\text{cm}$

$$\frac{1}{f'} = \frac{1}{2}\left(\frac{1}{5} - \frac{1}{-5}\right) = \frac{1}{5}\text{cm}$$

Now,

$$\frac{1}{f''} = \frac{1}{2}\left(\frac{1}{6} - \frac{1}{-6}\right) = \frac{2}{12} = \frac{1}{6}\text{ cm}$$

∴ $P' = \frac{1}{f'} = 0.2 \times 10^2 = 20\text{ D}$

$P'' = \frac{1}{f''} = 0.1667 \times 10^2 = 16.67\text{ D}$

∴ $P' - P'' = 20 - 16.67 = 3.33\text{ D}$

44. (B)

The angular acceleration of both the hour and second hands is zero, as they both move with constant angular velocity. Therefore, the ratio of their angular accelerations is 0.

45. (B)

$$v_{rms} = \sqrt{\frac{3RT}{M}} \Rightarrow v_{rms} \, \alpha \sqrt{T}$$

rms velocity is independent of pressure.

Therefore, the rms velocity will be twice the original velocity i.e. 2v

46. (B)

As the process is carried out in vacuum and in insulated surroundings, it is an adiabatic process and the pressure acting is zero. This indicates that the work done in the process is also zero. Thus, the internal energy of the system remains same. Therefore, temperature of the system does not change.

47. (A)

The fundamental frequency of a vibrating string

$$n = \frac{1}{2lr}\sqrt{\frac{T}{\pi\rho}} \qquad \Rightarrow n \propto \frac{1}{lr}$$

∴ $\frac{n_2}{n_1} = \frac{l_1 r_1}{l_2 r_2} = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$

∴ $n_2 = \frac{n_1}{4}$

48. (D)

Work done $W = pE[\cos\theta_0 - \cos\theta]$

Given, $W = pE(1 - \cos 180°)$

∴ $W = pE(1 - \cos 180°)$

$W = pE[1 - (-1)] = 2pE$

$pE = \frac{1}{2}W$

∴ When $\theta = 60°$

$W' = pE(1 - \cos 60°)$

$W' = pE\left(1-\frac{1}{2}\right)$

$W' = \frac{1}{2}\,pE$

$W' = \frac{1}{2}\times\frac{1}{2}\,W$

$W' = \frac{1}{4}\,W$

49. (D)

$R_{total} = \frac{GS}{G+S}$

$\frac{G}{30} = \frac{GS}{G+S}$

$30\,GS = G^2 + GS$

$29\,GS = G^2$

Dividing both side by 29 G

$S = \frac{G}{29}$ …(i)

Current flowing through shunt,

$I_S = I\cdot\frac{G}{G+S}$

$I_S = I\cdot\frac{G}{G+\frac{G}{29}}$ …[From(i)]

$I_S = I\frac{G}{G\left(\frac{30}{29}\right)} = I\left(\frac{29}{30}\right) = 96.6\%$

50. (A)

Work done in blowing soap bubbles with radius R and R/3 is,

$W_1 = 8\pi R^2 T_1$

$W_2 = 8\pi\left(\frac{R}{3}\right)^2 T_2 = \left(\frac{8}{9}\right)\pi R^2 T_2$

∴ $\frac{W_1}{W_2} = \frac{9T_1}{T_2}$

At $T_1 = T_2$, $W_1 = 9W_2$

When temperature decreases, surface tension increases.

∴ $W_1 > 9W_2$

51. (C)

Gases like NH_3 and CO_2 react with water as follows:

$NH_3 + H_2O \rightleftharpoons NH_4^+ + OH^-$

$CO_2 + H_2O \rightleftharpoons H_2CO_3$

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52. (A)
Resonance effect is observed in a conjugated system of π electrons. The carbonyl oxygen (O) has no conjugation with a π-system, as the adjacent CH_3 group lacks π-electrons to participate in resonance. Hence, resonance effect is not observed in the structure given in option (A).

53. (C)
Rate = k[Concentration]

$$\text{Rate} = \frac{0.693}{t_{1/2}}\,[\text{Concentration}]$$

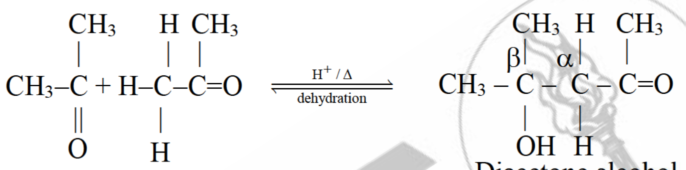
$$\therefore \quad t_{1/2} = \frac{0.693[\text{Concentration}]}{\text{Rate}}$$

$$t_{1/2} = \frac{0.693 \times 0.1}{0.69 \times 10^{-2}} = 10 \text{ min} = 600 \text{ s}$$

54. (B)
Conjugate base of an acid always has one fewer proton.

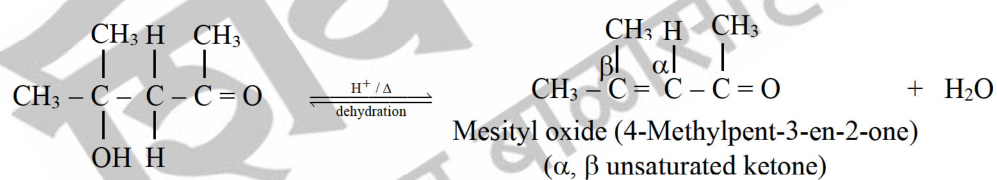
$$\text{Acid} \xrightarrow{-H^+} \text{Conjugate base}$$
$$H_2O \xrightarrow{-H^+} OH^-$$
$$HF \xrightarrow{-H^+} F^-$$

55. (A)



Acetone (2 moles)

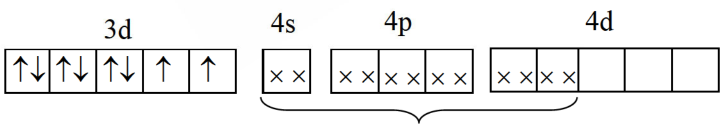
Diacetone alcohol
(4-Hydroxy-4-methylpentan-2-one)
(β-Hydroxy ketone)



Diacetone alcohol

Mesityl oxide (4-Methylpent-3-en-2-one)
(α, β unsaturated ketone)

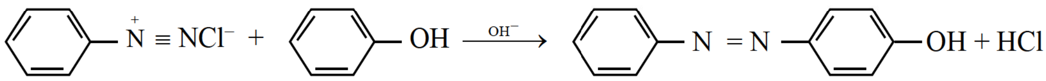
56. (D)
$[Ni(NH_3)_6]^{2+}$
Oxidation state of Ni = +2 ($3d^8$)



sp^3d^2 (outer)

57. (C)

58. (B)
Only option (B) involves retention of diazo group (azo coupling reaction).



Benzenediazonium chloride

Phenol

p-Hydroxyazobenzene (orange)

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59. (D)

Oxoacids	Oxidation state of S
H_2SO_4	+6
H_2SO_3	+4
$H_2S_2O_7$	+6
H_2S	–2

60. (D)

Molecules	Hybridization	s-character
NH_3	sp^3	25 %
CO_2	sp	50 %
H_2O	sp^3	25 %
PCl_5	sp^3d	20 %

Since PCl_5 has sp^3d hybridization, it has the lowest s-character (20%) among the given options.

61. (D)
Number of atoms in 0.25 mol = $0.25 \times N_A$
$= 0.25 \times 6.022 \times 10^{23} = 1.505 \times 10^{23}$
Number of tetrahedral voids
$= 2 \times$ Number of atoms $= 2 \times 1.505 \times 10^{23}$
$= 3.01 \times 10^{23}$

62. (A)
$\Delta G° = -2.303RT \log_{10} K_p$
$\therefore \Delta G° = -2.303 \times 8.314 \text{ J K}^{-1} \text{mol}^{-1} \times 298 \text{ K} \times \log_{10} (8 \times 10^{-6})$
$= -2.303 \times 8.314 \text{ J K}^{-1} \text{mol}^{-1} \times 298 \text{ K} \times (-5.097)$
$= 2.908 \times 10^4 \text{ J}$

63. (B)
Larger the value of E_a smaller is the rate constant.
$k' = 1.5 k''$ (given)
$\therefore k' > k''$
$\therefore E'_a < E''_a$

64. (C)
The metal ion exercises primary valences towards the negative groups to satisfy its normal charge by the formation of simple salts. It exercises secondary valences towards the negative ions, neutral molecules or both to form a coordination sphere.

65. (B)

66. (B)
In presence of peroxide, Anti-Markovnikov's product is the major product formed.

$$\underset{\text{Phenylethene}}{Ph–CH = CH_2} + HBr \xrightarrow[\text{Anti-Markovnikov's rule}]{H_2O_2} \underset{\text{1-Bromo-2-phenylethane (X)}}{Ph(CH_2)_2–Br}$$

67. (B)
The straight-chain structure of saturated fatty acids allows them to pack closely together, creating strong forces between the molecules. These strong van der Waals forces make saturated fats solid at room temperature.

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68. (C)

$P_1 = P$, $V_1 = V$, $T_1 = 273 + 77 = 350$ K

$P_2 = 3P$, $V_2 = \frac{65}{100}$

$\therefore \quad \frac{P \times V}{350} = \frac{3P \times 65V}{T_2 \times 100}$

$T_2 = \frac{350\ K \times 3 \times 65}{100}$

$\therefore \quad T_2 = 682.5$ K = 409.5 °C

69. (C)

$H_2O \longrightarrow 2H^+ + 2e^- + \frac{1}{2}O_2$

For the oxidation of one mole of water to dioxygen, 2 mole electrons are required.

Electricity required (Q) = Moles of electrons actually passed × 96500 C / mol e^-

$\therefore$ Q = 2 mol e^- × 96500 C / mol e^- = 1.93×10^5 C

70. (C)

71. (B)

72. (C)

Volume occupied by 1 mole of any gas at STP = 22.4 dm^3

$\therefore$ Volume occupied by 11.2g of N_2 i.e., 0.4 mole of N_2 at STP = 0.4 × 22.4 dm^3 = 8.96 L

73. (C)

According to the Freundlich adsorption isotherm:

$\frac{x}{m} \propto P^{1/n}$

$\therefore \quad \log \frac{x}{m} = \log k + \frac{1}{n} \log P$

This corresponds to the equation of a straight line: $y = mx + c$

$\therefore$ Slope of the line (m) is equal to $\frac{1}{n} = \frac{2}{3}$

$\therefore \quad \frac{x}{m} \propto P^{2/3}$

74. (A)

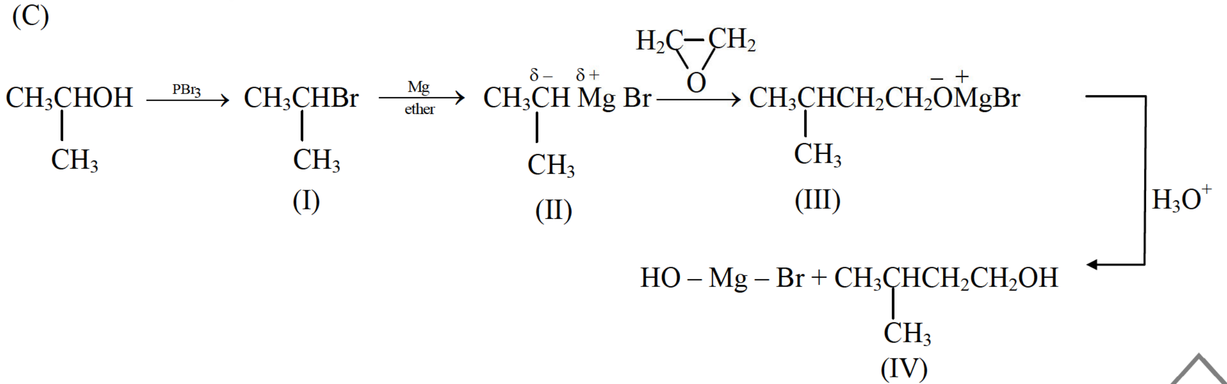
For $Ba(NO_3)_2$, i = 2.74 ….(Given)

n = 3

Degree of dissociation (α)

$= \frac{i-1}{n-1} = \frac{2.74-1}{3-1} = 0.87$

75. (C)



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76. (C)

77. (C)

Let the rate law for the reaction be,

Rate = k $[A]^x [B]^y$

$0.075 = k [0.05]^x [0.05]^y$ ...(i)

$0.15 = k [0.10]^x [0.05]^y$ ...(ii)

$1.2 = k [0.20]^x [0.10]^y$ ...(iii)

Dividing equation (ii) by (i),

$$\frac{0.15}{0.075} = \left(\frac{0.10}{0.05}\right)^x$$

$2^1 = 2^x$

∴ $x = 1$

Dividing equation (iii) by (i),

$$\frac{1.2}{0.075} = \frac{(0.20)^x (0.10)^y}{(0.05)^x (0.05)^y} = \left(\frac{0.2}{0.05}\right)^1 \left(\frac{0.10}{0.05}\right)^y$$

$16 = 4 \times 2^y$

$2^y = 4$

$2^y = 2^2$

∴ $y = 2$

Thus, rate = k [A] $[B]^2$

78. (D)

Expression for work, when an ideal gas expands isothermally and reversibly is given by

$W_{max} = -2.303nRT \log_{10} \frac{V_2}{V_1}$.

For same mass, temperature, V_1 and V_2

$W_{max} \propto n$;

where, n = number of moles = $\frac{\text{Mass}}{\text{Molar mass}}$

$n \propto \frac{1}{\text{Molar mass}}$

∴ Lower the molar mass, higher is the magnitude of work.

79. (D)

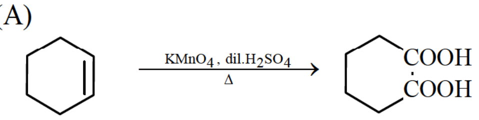
According to Bohr's theory, $r_n \propto n^2$

$r_1 = \gamma$, $n_1 = 1$, $n_4 = 4$

$$\frac{r_1}{r_4} = \left(\frac{1}{4}\right)^2 = \frac{1}{16}$$

∴ $r_4 = 16r_1 = 16\gamma$

80. (A)



Cyclohexene Adipic acid

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81. (A)

α-Amino acids are generally represented as, R–CH(NH_2)COOH (where R – Side chain)

If 'R' contains: –COOH group then the amino acid is acidic; an amino (1°, 2° or 3°) group then the amino acid is basic, or having neutral/no functional group in 'R' then the amino acid is neutral.

∴ Neutral α-amino acids: i, iii

Acidic α-amino acids: ii, v

Basic α-amino acids: iv

82. (C)

Ti : $[Ar]\ 3d^2\ 4s^2 \Rightarrow Ti^+ : [Ar]\ 3d^2\ 4s^1$

V : $[Ar]\ 3d^3\ 4s^2 \Rightarrow V^+ : [Ar]\ 3d^3\ 4s^1$

Cr : $[Ar]\ 3d^5\ 4s^1 \Rightarrow Cr^+ : [Ar]\ 3d^5\ 4s^0$

Mn : $[Ar]\ 3d^5\ 4s^2 \Rightarrow Mn^+ : [Ar]\ 3d^5\ 4s^1$

In Cr, for second ionization, the electron needs to be removed from stable half-filled orbital and hence requires more energy. Therefore, the order of decreasing second ionization enthalpy is Cr > Mn > V > Ti.

83. **(C)**

Due to poor shielding of 4f electrons (i.e., negligible screening effect of 'f' orbitals), increased effective nuclear charge is experienced and the valence shell is pulled slightly towards nucleus. This results in the lanthanoid contraction.

84. (B)

$$\underset{\underset{Br}{|}}{CH_3-CH-CH_3} + \underset{\text{(alcoholic)}}{KOH} \longrightarrow \underset{\text{(A)}}{CH_3-CH=CH_2} + KBr + H_2O$$

$$\underset{\text{(A)}}{CH_3-CH=CH_2} + HBr \xrightarrow{\text{Peroxide}} \underset{\substack{\text{1-Bromopropane}\\ \text{(B)}}}{CH_3-CH_2-CH_2-Br}$$

$$\underset{\text{(B)}}{CH_3-CH_2-CH_2-Br} + CH_3-O-Na \longrightarrow \underset{\substack{\text{Methyl n-propyl ether}\\ \text{(C)}}}{CH_3-CH_2-CH_2-O-CH_3} + NaBr$$

85. (A)

$\wedge_0\ (HA) = \wedge_0\ (HCl) + \wedge_0\ (NaA) - \wedge_0\ (NaCl)$

$= 425.9 + 100.5 - 126.4$

$\approx 400\ S\ cm^2\ mol^{-1}$

$$\wedge = \frac{1000k}{c}$$

$$\wedge = \frac{1000(cm^3\ L^{-1}) \times 5 \times 10^{-4}\ (S\ cm^{-1})}{0.004(mol\ L^{-1})}$$

$= 125\ S\ cm^2\ mol^{-1}$

$$\alpha = \frac{\wedge_c}{\wedge_0} = \frac{125\ S\ cm^2\ mol^{-1}}{400\ S\ cm^2\ mol^{-1}} = 0.31$$

86. (B)

$250\ cm^3 = 250 \times 10^{-6}\ m^3$

$W = -P_{ext}\Delta V$

$= -2 \times 10^5\ N\ m^{-2} \times (250 \times 10^{-6}\ m^3)$

$= -50\ N\ m$

$= -50\ J$

∴ $W = -50\ J$

Now,

∴ $\Delta U = Q + P_{ext}\Delta V$

$= 160\text{ J} + (-50\text{ J}) = 110\text{ J}$

87. (A)

N – H bonds in amines are less polar than O – H bond in alcohols. Thus, water solubilities of alcohols, amines and alkanes of comparable molar mass in water decreases in the order: alcohols > amines > alkanes.

88. (A)

Potassium (K) as an alkali metal, has only one valence electron, which requires less energy to remove compared to the two valence electrons in magnesium (Mg) which is an alkaline earth metal. This makes K more reactive. Mg, despite forming a divalent cation, requires more energy to lose two electrons, making it less reactive.

Reactivity is primarily influenced by ionization energy rather than atomic mass or s-block classification.

89. (B)

$2MnO_4^- + 5C_2O_4^{2-} + 16H^+ \longrightarrow 2Mn^{2+} + 10CO_2 + 8H_2O$

Ratio of $x : y = 2 : 5$

90. (D)

1.4 L = 1 g

22.4 L = 16 g

Hence, gas is methane (CH_4) and RMgBr is CH_3MgBr.

91. (C)

Let the number of atoms of element X in hcp unit cell be n.

∴ Number of tetrahedral voids = 2n

As 70% of the tetrahedral voids are occupied by atoms of element Y,

Number of atoms of element Y = 2n × 0.70 = 1.4n

∴ Ratio of atoms of element X to atoms of element Y = n : 1.4n = 5 : 7

Hence, the formula of the compound is X_5Y_7.

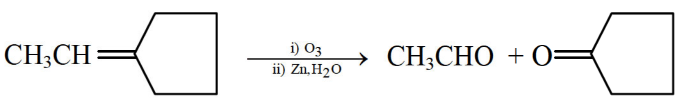
92. (B)

The carbonyl group of ketone is reduced to methylene group ($-CH_2-$) on treatment with zinc-amalgam and conc. HCl (Clemmensen reduction).

93. (A)

(A) $CH_2 = CH - CH_2 - CH = CH_2 \xrightarrow[\text{ii) Zn, }H_2O]{\text{i) }O_3} 2HCHO + OHC - CH_2 - CHO$

Penta-1,4-diene; Formaldehyde; Propanedial

(B) 

Acetaldehyde Cyclopentanone

(C) $HC \equiv CH + H_2O \xrightarrow[1\%\ HgSO_4]{40\%\ H_2SO_4} CH_3CHO$

Acetaldehyde

(D) $CH_3COCl + H_2 \xrightarrow{Pd\text{-}BaSO_4} CH_3CHO$

Acetaldehyde

94. (B)

$$\pi = \frac{W_2}{M_2} \times \frac{1}{V} RT$$

$\therefore$ $1.23 \text{ atm} = \frac{0.6\text{ g}}{M} \times \frac{1}{0.1\text{L}} \times 0.0821 \text{ L atm K}^{-1} \text{ mole}^{-1} \times 300 \text{ K}$

$\therefore$ $M_2 = \frac{0.6}{0.1} \times \frac{1}{1.23} \times 0.0821 \times 300 \text{ g/mole}$

$\therefore$ $M_2 = \frac{6}{12.3} \times 0.821 \times 300$

$= \frac{1 \times 82.1 \times 3}{2.05}$

$= 40.05 \times 3$

$\therefore$ $M_2 = 120.15 \text{ g/mol}$

95. (D)

pH = 12

pOH = 14 – 12 = 2

$[OH^-] = 1 \times 10^{-2} M$

$$Ba(OH)_{2(S)} \rightleftharpoons Ba^{2+}_{(aq)} + 2OH^-_{(aq)}$$

$$S \qquad 2S = 1 \times 10^{-2} M$$

Since, $2S = 1 \times 10^{-2}$, $S = 0.5 \times 10^{-2}$ M

Now,

$K_{sp} = [Ba^{2+}][OH^-]^2 = S \times (2S)^2$

$= 0.5 \times 10^{-2} (1 \times 10^{-2})^2$

$= 0.5 \times 10^{-6}$

96. (A)

Lower is the reduction potential, greater is the reducing power. Hence, the increasing order of reducing power is Au < Pb < Mg.

97. (D)

Oxidation of alkenes with hot, concentrated $KMnO_4$ leads to cleavage of the double bond, forming carboxylic acids (for terminal alkenes) or ketones (for internal alkenes).

98. (B)

99. (B)

This is due to a common ion effect. Three chloride ions (Cl^-) will be produced per molecule of $AlCl_3$ (which is maximum amongst given options).

100. (C)

101. (A)

$$\lim_{x\to 1}\left[\frac{x^2+x\sqrt{x}-2}{x-1}\right] = \lim_{x\to 1}\left[\frac{(x^2-1)+(x\sqrt{x}-1)}{x-1}\right]$$

$$= \lim_{x\to 1}\left[\frac{x^2-1}{x-1}+\frac{x^{\frac{3}{2}}-1}{x-1}\right] \quad \ldots\left[\because x\sqrt{x} = x^1\cdot x^{\frac{1}{2}} = x^{1+\frac{1}{2}} = x^{\frac{3}{2}}\right]$$

$$= \lim_{x\to 1}\left(\frac{x^2-1^2}{x-1}\right) + \lim_{x\to 1}\left(\frac{x^{\frac{3}{2}}-1^{\frac{3}{2}}}{x-1}\right)$$

$$= 2(1)^1 + \frac{3}{2}(1)^{\frac{1}{2}} \quad \ldots\left[\because \lim_{x\to a}\frac{x^n-a^n}{x-a} = na^{n-1}\right]$$

$$= 2+\frac{3}{2} = \frac{7}{2}$$

102. (A)

The given equation is defined for $x \neq \frac{\pi}{2}, \frac{3\pi}{2}$.

Now, $\tan x + \sec x = 2\cos x$

$\Rightarrow \frac{\sin x}{\cos x} + \frac{1}{\cos x} = 2\cos x$

$\Rightarrow (\sin x + 1) = 2\cos^2 x$

$\Rightarrow (\sin x + 1) = 2(1 - \sin^2 x)$

$\Rightarrow (\sin x + 1) = 2(1 - \sin x)(1 + \sin x)$

$\Rightarrow (1 + \sin x)[2(1 - \sin x) - 1] = 0$

$\Rightarrow 2(1 - \sin x) - 1 = 0 \quad \ldots\left[\because \sin x \neq -1 \text{ otherwise } \cos x = 0 \text{ and } \tan x, \sec x \text{ will be undefined}\right]$

$\Rightarrow \sin x = \frac{1}{2}$

$\Rightarrow x = \frac{\pi}{6}, \frac{5\pi}{6}$ in $(0, 2\pi)$

103. (A)

$y = (x\log x)^{\log(\log x)}$

Taking logarithm on both sides, we get

$\log y = \log(\log x)[\log x + \log(\log x)]$

Differentiating w.r.t. x, we get

$$\frac{1}{y}\cdot\frac{dy}{dx} = \frac{1}{x\log x}[\log x + \log(\log x)] + \log(\log x)\left(\frac{1}{x}+\frac{1}{x\log x}\right)$$

$$\Rightarrow \frac{dy}{dx} = (x\log x)^{\log(\log x)}\left\{\frac{1}{x\log x}[\log x + \log(\log x)] + \log(\log x)\left(\frac{1}{x}+\frac{1}{x\log x}\right)\right\}$$

104. (C)

Let $I = \int\frac{x^3\sin\left[\tan^{-1}(x^4)\right]}{1+x^8}dx$

Put $x^4 = t \Rightarrow 4x^3\,dx = dt$

$\therefore \quad I = \frac{1}{4}\int\frac{\sin(\tan^{-1}t)}{1+t^2}dt$

Put $\tan^{-1} t = z \Rightarrow \frac{1}{1+t^2}dt = dz$

$\therefore \quad I = \frac{1}{4}\int \sin z\, dz = \frac{1}{4}(-\cos z) + c$

$= -\frac{1}{4}\cos(\tan^{-1}t) + c$

$= \frac{-1}{4}\cos[\tan^{-1}(x^4)] + c$

105. (D)

$\frac{dy}{dx} = -\frac{x-2y+1}{2(x-2y)+3}$ ...(i)

Put $x - 2y = v$ ...(ii)

$\Rightarrow 1 - \frac{2dy}{dx} = \frac{dv}{dx}$

$\Rightarrow \frac{dy}{dx} = \frac{1}{2}\left(1 - \frac{dv}{dx}\right)$ ...(iii)

Substituting (ii) and (iii) in (i), we get

$\frac{1}{2}\left(1 - \frac{dv}{dx}\right) = -\frac{v+1}{2v+3}$

$\Rightarrow \frac{2v+3}{4v+5}\, dv = dx$

Integrating on both sides, we get

$\int \left[\frac{\frac{1}{2}(4v+5) + \frac{1}{2}}{4v+5}\right] dv = \int dx + c_1$

$\Rightarrow \frac{1}{2}v + \frac{1}{2}\cdot\frac{1}{4}\log(4v+5) = x + c_1$

$\Rightarrow \frac{1}{2}(x-2y) + \frac{1}{8}\log[4(x-2y)+5] = x + c_1$

$\Rightarrow \log[4(x-2y)+5] = 8x - 4(x-2y) + 8c_1$

$\Rightarrow \log[4(x-2y)+5] = 4(x+2y) + c$, where $c = 8c_1$

106. (A)

$16x^2 - 3y^2 - 32x - 12y - 44 = 0$

$\Rightarrow 16(x^2 - 2x) - 3(y^2 + 4y) - 44 = 0$

$\Rightarrow 16(x-1)^2 - 3(y+2)^2 = 48$

$\Rightarrow \frac{(x-1)^2}{3} - \frac{(y+2)^2}{16} = 1$

Comparing with standard form, we get

$a^2 = 3, b^2 = 16$

$e = \sqrt{1+\frac{b^2}{a^2}} = \sqrt{1+\frac{16}{3}} = \sqrt{\frac{19}{3}}$

107. (A)

$\cos 2B = \frac{\cos(A+C)}{\cos(A-C)}$

$\frac{1-\tan^2 B}{1+\tan^2 B} = \frac{\cos A\cos C - \sin A\sin C}{\cos A\cos C + \sin A\sin C}$

$$\frac{1-\tan^2 B}{1+\tan^2 B} = \frac{\cos A \cos C(1-\tan A \tan C)}{\cos A \cos C(1+\tan A \tan C)}$$

$$\frac{1-\tan^2 B}{1+\tan^2 B} = \frac{(1-\tan A \tan C)}{(1+\tan A \tan C)}$$

$(1 - \tan^2B)(1 + \tan A \tan C) = (1 - \tan A \tan C)(1 + \tan^2B)$

$1 + \tan A \tan C - \tan^2B - \tan^2B \tan A \tan C = 1 + \tan^2B - \tan A \tan C - \tan A \tan C \tan^2B$

$2\tan A \tan C = 2\tan^2B$

$\tan^2B = \tan A \cdot \tan C$

$\therefore$ tanA, tanB, tanC are in G.P.

108. (A)

$$\bar{a}\cdot\bar{b} = |\bar{a}||\bar{b}|\cos 60^\circ = (2)(3)\left(\frac{1}{2}\right) = 3$$

$$\bar{b}\cdot\bar{c} = |\bar{b}||\bar{c}|\cos 60^\circ = (3)(5)\left(\frac{1}{2}\right) = \frac{15}{2}$$

$$\bar{a}\cdot\bar{c} = |\bar{a}||\bar{c}|\cos 60^\circ = (2)(5)\left(\frac{1}{2}\right) = 5$$

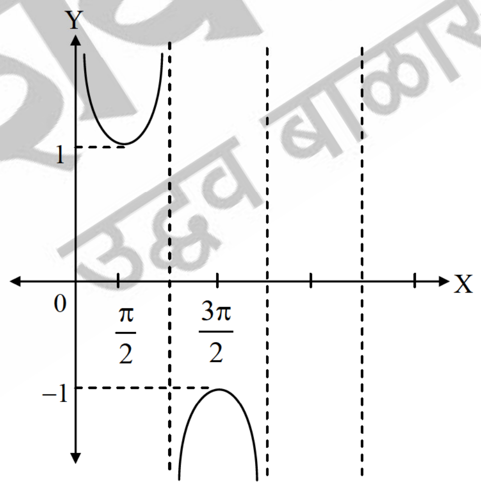
$\therefore$ $$|\bar{a}+\bar{b}+\bar{c}|^2 = |\bar{a}|^2 + |\bar{b}|^2 + |\bar{c}|^2 + 2(\bar{a}\cdot\bar{b} + \bar{b}\cdot\bar{c} + \bar{c}\cdot\bar{a})$$

$$= 2^2 + 3^2 + 5^2 + 2\left(3 + \frac{15}{2} + 5\right) = 4 + 9 + 25 + 31 = 69$$

$\therefore$ $$|\bar{a}+\bar{b}+\bar{c}| = \sqrt{69}$$

109. (A)

The graph of cosec x is opposite in interval $\left(\frac{\pi}{2}, \frac{3\pi}{2}\right)$



At the point $x = \pi$, cosec x is not defined and $x \in \left(\frac{\pi}{2}, \frac{3\pi}{2}\right)$

$\therefore$ equation is neither increasing nor decreasing.

Also, $\frac{d}{dx}(\tan x) = \sec^2 x > 0$ which is a increasing function.

Also $y = x^2$ is a parabola, which is increasing

Also $y = |x - 1|$ is a V-shaped upward curve, which is always increasing.

$\therefore$ option (A) is the correct answer.

110. (B)

$$P(X) = \begin{cases} \dfrac{2x}{n(n+1)}, & x = 1,2,3,....,n \\ 0, & \text{othewise} \end{cases}$$

$\therefore$ $E(X) = \sum_{i=1}^{n} x_i p(x_i)$

$$= \frac{2}{n(n+1)} + \frac{8}{n(n+1)} + ... + \frac{2n^2}{n(n+1)}$$

$$= \frac{2(1^2 + 2^2 + ... + n^2)}{n(n+1)}$$

$$= \frac{2n(n+1)(2n+1)}{6n(n+1)} = \frac{2n+1}{3}$$

111. (A)

Let $I = \int e^{x^2} \cdot x^3 \, dx$

Put $x^2 = t \Rightarrow 2x\,dx = dt$

$$I = \int e^t \cdot t\left(\frac{dt}{2}\right)$$

$$= \frac{1}{2}\int t \cdot e^t dt = \frac{1}{2}(te^t - e^t) + C = \frac{e^{x^2}}{2}(x^2 - 1) + C$$

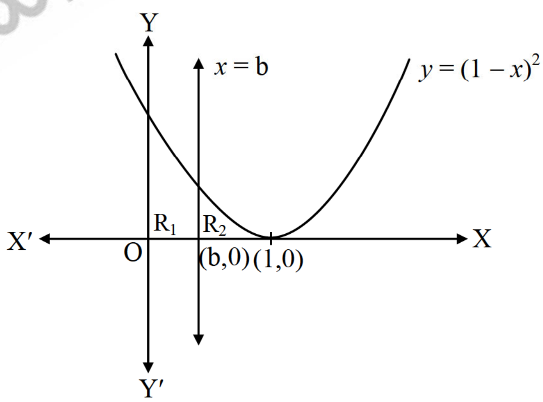
Comparing with $e^{x^2} f(x) + C$, we get

$$f(x) = \frac{x^2 - 1}{2}$$

$\therefore$ $f(2) = \frac{3}{2}$

112. (B)

Here, $R_1 = \int_0^b (1-x)^2 dx$ and $R_2 = \int_b^1 (1-x)^2 dx$



$\therefore$ $R_1 = \left[\frac{(x-1)^3}{3}\right]_0^b$ and $R_2 = \left[\frac{(x-1)^3}{3}\right]_b^1$

$\Rightarrow R_1 = \frac{(b-1)^3}{3} + \frac{1}{3}$ and $R_2 = -\frac{(b-1)^3}{3}$

Since $R_1 - R_2 = \frac{1}{4}$

$\therefore$ $\frac{(b-1)^3}{3} + \frac{1}{3} + \frac{(b-1)^3}{3} = \frac{1}{4}$

$\Rightarrow \frac{2}{3}(b-1)^3 = -\frac{1}{12}$

$\Rightarrow (b-1)^3 = -\frac{1}{8}$

$\Rightarrow b - 1 = -\frac{1}{2}$

$\Rightarrow b = \frac{1}{2}$

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113. (C)

Consider option (C)

$$\left(3\hat{i}+2\hat{j}+6\hat{k}\right)\cdot\left(\frac{-8\hat{i}+3\hat{j}+3\hat{k}}{\sqrt{82}}\right)=0$$

This is valid for only option (C)

$\therefore$ Option (C) is correct.

114. (C)

$$\frac{2z+1}{iz+1}=\frac{2(x+iy)+1}{i(x+iy)+1}$$

$$=\frac{2x+1+2iy}{1-y+ix}$$

$$=\frac{(2x+1+2iy)(1-y-ix)}{(1-y+ix)(1-y-ix)}$$

$$=\frac{(2x+1+2iy)(1-y-ix)}{(1-y)^2+x^2}$$

Imaginary part $=\dfrac{2y(1-y)-x(2x+1)}{(1-y)^2+x^2}$

$\Rightarrow -2=\dfrac{2y-2y^2-2x^2-x}{1-2y+y^2+x^2}$

$\Rightarrow -2\ (1-2y+y^2+x^2)=2y-2y^2-2x^2-x$

$\Rightarrow 6y-x=-2,$

which is a straight line.

115. (D)

$(A-2I)\ (A-4I)=0$

$\Rightarrow A^2-4AI-2AI+8I=0$

$\Rightarrow A^2-6A+8I=0$

Multiplying by A^{-1}, we get

$A-6I+8A^{-1}=0$

$\Rightarrow A+8A^{-1}=6I$

$\Rightarrow \dfrac{1}{6}A+\dfrac{4}{3}A^{-1}=I$

116. (B)

$\cos^{-1}x+\cos^{-1}y+\cos^{-1}z=\pi$ ...[Given]

$\therefore$ $\cos^{-1}x+\cos^{-1}y=\pi-\cos^{-1}z$

$\therefore$ $\cos^{-1}x+\cos^{-1}y=\cos^{-1}(-z)$ ...[$\because \cos^{-1}(-\theta)=\pi-\cos^{-1}\theta$]

Let, $\cos^{-1}x=M$ and $\cos^{-1}y=N$

$\therefore$ $M+N=\cos^{-1}(-z)$ ...(i)

Also, $x=\cos M$ and $y=\cos N$ ...(ii)

$\therefore$ $\sin M=\sqrt{1-\cos^2 M}$ and $\sin N=\sqrt{1-\cos^2 M}$ ...[$\because \sin^2\theta+\cos^2\theta=1$]

$\therefore$ $\sin M=\sqrt{1-x^2}$ and $\sin N=\sqrt{1-y^2}$ ...(iii)

Consider $\cos(M+N)=\cos M\cos N-\sin M\sin N$

$\therefore$ $\cos(M+N)=xy-\sqrt{1-x^2}\sqrt{1-y^2}$ ...[From (ii) and (iii)]

$\therefore$ $M+N=\cos^{-1}\left(xy-\sqrt{1-x^2}\sqrt{1-y^2}\right)$

$\therefore$ $\cos^{-1}(-z)=\cos^{-1}\left(xy-\sqrt{1-x^2}\sqrt{1-y^2}\right)$ ...[From (i)]

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Equating both sides, we get

$-z = xy - \sqrt{1-x^2}\ \sqrt{1-y^2}$

$\therefore \quad \sqrt{1-x^2}\ \sqrt{1-y^2} = xy + z$

By taking square of both the sides, we get

$(1-x^2)(1-y^2) = (xy+z)^2$

$\therefore \quad 1 - y^2 - x^2 + x^2y^2 = (xy)^2 + 2xyz + z^2$

$\therefore \quad 1 - y^2 - x^2 + x^2y^2 = x^2y^2 + 2xyz + z^2$

$\therefore \quad 1 - y^2 - x^2 = 2xyz + z^2$

$\therefore \quad 1 = 2xyz + z^2 + y^2 + x^2$

$\therefore \quad x^2 + y^2 + z^2 = 1 - 2xyz$

117. (B)

Let OA and OB be two lines through the origin, each making an angle of 30° with the line $3x + 2y - 11 = 0$.
Let slope of OA (or OB) be m.

Slope of the line $3x + 2y - 11 = 0$ is $-\frac{3}{2}$ and $\theta = 30°$

$\therefore \quad \tan 30° = \left|\frac{m - \left(-\frac{3}{2}\right)}{1 + \left(-\frac{3}{2}\right)m}\right|$

$\therefore \quad \frac{1}{\sqrt{3}} = \left|\frac{2m+3}{2-3m}\right|$

By taking square of both sides, we get

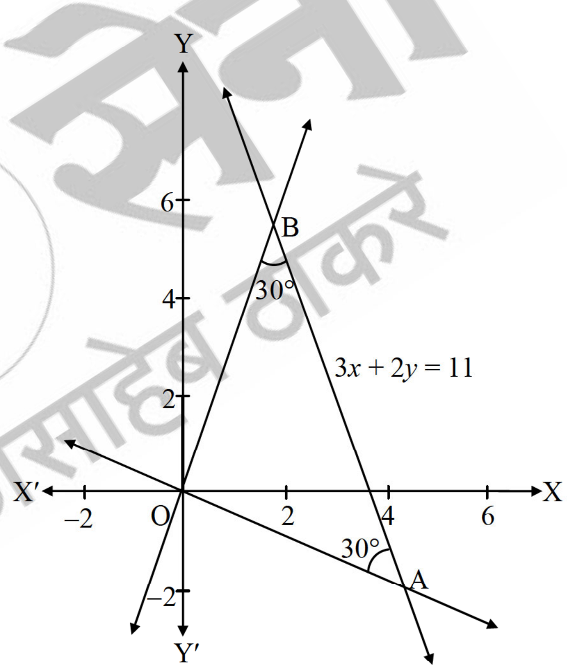
$(2-3m)^2 = 3(2m+3)^2$

$\therefore \quad 4 - 12m + 9m^2 = 12m^2 + 36m + 27$

$\therefore \quad 3m^2 + 48m + 23 = 0 \quad \ldots(i)$

Equation of OA (or OB) is $y = mx$, since it passes through the origin.

$\therefore \quad m = \frac{y}{x}$



Substituting the value of m in (i), we get

$3\left(\frac{y}{x}\right)^2 + 48\left(\frac{y}{x}\right) + 23 = 0$

$\therefore \quad \frac{3y^2}{x^2} + \frac{48y}{x} + 23 = 0$

$\therefore \quad 3y^2 + 48xy + 23x^2 = 0$

$\therefore \quad 23x^2 + 48xy + 3y^2 = 0$ is the required combined equation.

118. (C)

$f(x) = \log x$

$\Rightarrow f'(x) = \frac{1}{x}$

By Lagrange's Mean value theorem,

$f'(c) = \frac{f(e) - f(1)}{e - 1}$

$\Rightarrow \frac{1}{c} = \frac{\log e - \log 1}{e - 1}$

$\Rightarrow \frac{1}{c} = \frac{1}{e-1}$

$\Rightarrow c = e - 1$

119. (A)

Let $I = \int_{\frac{-1}{2}}^{\frac{1}{2}} \left([x] + \log_e\left(\frac{1+x}{1-x}\right)\right) dx$

$= \int_{\frac{-1}{2}}^{0} [x]\, dx + \int_{0}^{\frac{1}{2}} [x]\, dx + \int_{\frac{-1}{2}}^{\frac{1}{2}} \log_e\left(\frac{1+x}{1-x}\right) dx$

Let $g(x) = \log\left(\frac{1+x}{1-x}\right)$

$g(-x) = \log\left(\frac{1-x}{1+x}\right)$

$= -\log\left(\frac{1+x}{1-x}\right) = -g(x)$

∴ $g(x)$ is a odd function.

∴ $\int_{\frac{-1}{2}}^{\frac{1}{2}} g(x)\, dx = 0$

∴ $I = \int_{\frac{-1}{2}}^{0} (-1)\, dx + \int_{0}^{\frac{1}{2}} (0)\, dx + 0$

$= [-x]_{\frac{-1}{2}}^{0} + 0 = \frac{-1}{2}$

120. (C)

Let ‘θ’ be the temperature of the body at any time ‘t’.

∴ $\frac{d\theta}{dt} \propto (\theta - 20)$

∴ $\frac{d\theta}{dt} = k(\theta - 20)$

Integrating on both sides, we get

$\log(\theta - 20) = kt + c$

When $t = 0$, $\theta = 100°$ C

∴ $\log(100 - 20) = k(0) + c \Rightarrow c = \log 80$

∴ $\log(\theta - 20) = kt + \log 80$ …(i)

When $t = 20$, $\theta = 60°$ C

∴ $\log(60 - 20) = k(20) + \log 80$

$\Rightarrow k = \frac{1}{20} \log\left(\frac{1}{2}\right)$

∴ $\log(\theta - 20) = \frac{t}{20} \log\left(\frac{1}{2}\right) + \log 80$ …[From (i)]

When $\theta = 30°$ C, we have

$\log(30 - 20) = t\left(\frac{1}{20}\right) \log\left(\frac{1}{2}\right) + \log 80$

$\Rightarrow \log 10 - \log 80 = \frac{t}{20} \log\left(\frac{1}{2}\right)$

$\Rightarrow \log\left(\frac{1}{8}\right) = \frac{t}{20}\log\left(\frac{1}{2}\right)$

$\Rightarrow 3\log\left(\frac{1}{2}\right) = \frac{t}{20}\log\left(\frac{1}{2}\right)$

$\Rightarrow \frac{t}{20} = 3$

$\Rightarrow t = 60$ minutes

121. (C)

$x^2 + y^2 - 6x - 4y - 12 = 0$

$\therefore$ $(x^2 - 6x + 9 - 9) + (y^2 - 4y + 4 - 4) - 12 = 0$

$\therefore$ $(x - 3)^2 + (y - 2)^2 = 25$

$\therefore$ for circle C_1 : Centre is (3, 2) and radius = 5

$\therefore$ Area of $C_1 = \pi r^2 = 25\pi$

Let the radius of required circle be R.

Area of required circle = 4 (Area of C_1)

$\therefore$ $\pi R^2 = 4(25\pi)$

$\therefore$ $R^2 = 100$

$\therefore$ R = 10 units

$\therefore$ Equation of the required circle is $(x - 3)^2 + (y - 2)^2 = 100$

$\therefore$ $x^2 + y^2 - 6x - 4y = 87$

$\therefore$ $x^2 + y^2 - 6x - 4y - 87 = 0$

122. (C)

Let p : The woman in a family is literate.

q : Family becomes literate.

$\therefore$ The symbolic form of Statement I is $p \to q$, and

The symbolic form of statement II is $q \to p$

$\therefore$ Statement II is converse of statement I.

123. (B)

Let $\bar{a}$, $\bar{b}$, $\bar{c}$, $\bar{0}$ be the position vectors of points A, B, C, D respectively.

Consider

$\left|\overline{AB} \times \overline{CD} + \overline{BC} \times \overline{AD} + \overline{CA} \times \overline{BD}\right|$

$= \left|\left(\bar{b} - \bar{a}\right) \times \left(\bar{0} - \bar{c}\right) + \left(\bar{c} - \bar{b}\right) \times \left(\bar{0} - \bar{a}\right) + \left(\bar{a} - \bar{c}\right) \times \left(\bar{0} - \bar{b}\right)\right|$

$= \left|\left(\bar{b} - \bar{a}\right) \times \left(-\bar{c}\right) + \left(\bar{c} - \bar{b}\right) \times \left(-\bar{a}\right) + \left(\bar{a} - \bar{c}\right) \times \left(-\bar{b}\right)\right|$

$= \left|-\bar{b} \times \bar{c} + \bar{a} \times \bar{c} - \bar{c} \times \bar{a} + \bar{b} \times \bar{a} - \bar{a} \times \bar{b} + \bar{c} \times \bar{b}\right|$

$= \left|-\bar{b} \times \bar{c} - \bar{c} \times \bar{a} - \bar{c} \times \bar{a} - \bar{a} \times \bar{b} - \bar{a} \times \bar{b} - \bar{b} \times \bar{c}\right|$

$= \left|-2\left(\bar{a} \times \bar{b} + \bar{b} \times \bar{c} + \bar{c} \times \bar{a}\right)\right|$

$= 2\left|\bar{a} \times \bar{b} + \bar{b} \times \bar{c} + \bar{c} \times \bar{a}\right|$ …(i)

Area of $\Delta ABC = \frac{1}{2}\left|\overline{AB} \times \overline{AC}\right|$

$= \frac{1}{2}\left|\left(\bar{b} - \bar{a}\right) \times \left(\bar{c} - \bar{a}\right)\right|$

$= \frac{1}{2}\left|\bar{b} \times \bar{c} - \bar{b} \times \bar{a} - \bar{a} \times \bar{c} + \bar{a} \times \bar{a}\right|$

$\therefore$ Area of $\Delta ABC = \frac{1}{2}\left|\bar{b}\times\bar{c}+\bar{a}\times\bar{b}+\bar{c}\times\bar{a}+0\right|$

$\therefore$ 2 (Area of ΔABC) $= \left|\bar{a}\times\bar{b}+\bar{b}\times\bar{c}+\bar{c}\times\bar{a}\right|$ …(ii)

$\therefore$ $\left|\overline{AB}\times\overline{CD}+\overline{BC}\times\overline{AD}+\overline{CA}\times\overline{BD}\right| = 2\times 2$ (Area of ΔABC) …[From (i) and (ii)]

$= 4$(Area of ΔABC)

$\therefore$ $\lambda = 4$

124. (B)

Since $x = 1$ and $x = 3$ are extreme points of $p(x)$.

$\therefore$ $p'(1) = 0$ and $p'(3) = 0$

$\therefore$ $(x-1)$ and $(x-3)$ are the factors of $p'(x)$.

$\therefore$ $p'(x) = k(x-1)(x-3) = k(x^2 - 4x + 3)$

$\Rightarrow p(x) = k\left(\frac{x^3}{3} - 2x^2 + 3x\right) + c$

Given, $p(1) = 6$ and $p(3) = 2$

$\Rightarrow 6 = k\left(\frac{1}{3} - 2 + 3\right) + c$ and $2 = k(9-18+9) + c$

$\Rightarrow 6 = \frac{4k}{3} + c$ and $c = 2 \Rightarrow k = 3$

$\therefore$ $p'(x) = 3(x^2 - 4x + 3)$

$p'(0) = 9$

125. (A)

The probability that ship 'A' reaches safely is $P(A) = \frac{2}{2+5} = \frac{2}{7}$

The probability that ship 'B' reaches safely is $P(B) = \frac{3}{3+7} = \frac{3}{10}$

The probability that ship 'C' reaches safely is $P(C) = \frac{6}{6+11} = \frac{6}{17}$

$\therefore$ Probability that all of them arriving safely $= P(A \cap B \cap C)$

$= P(A) \cdot P(B) \cdot P(C)$ [Since A, B, C are all independent events]

$= \frac{2}{7}\times\frac{3}{10}\times\frac{6}{17} = \frac{18}{595}$

126. (A)

Let the corner points of the feasible region be A, B, C, D.

Solving equations $y = 3$ and $2x + 3y = 12$, we get

A = (1.5, 3)

Similarly,

B = (4, 3), C = (4, 7), D = $\left(\frac{3}{5}, \frac{18}{5}\right)$

Let $Z = 3x + 5y$

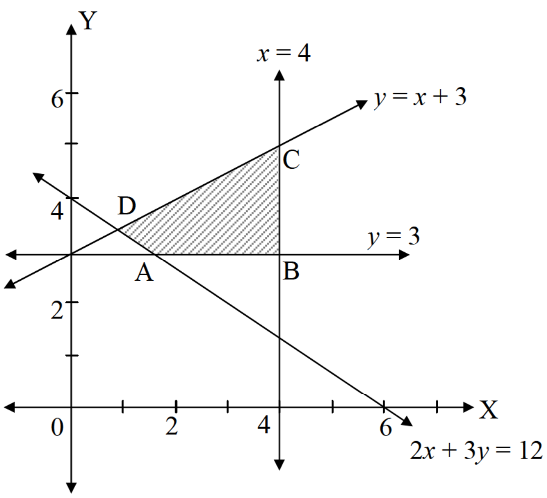
$\therefore$ Value of Z at point A = 19.5

Value of Z at point B = 27

Value of Z at point C = 47

Value of Z at point D = $\frac{99}{5}$

$\therefore$ The minimum value of Z is 19.5.



127. (D)

Let $I = \int \frac{2e^{x} + 3e^{-x}}{3e^{x} + 4e^{-x}}\, dx$

$= \int \frac{2e^{2x} + 3}{3e^{2x} + 4}\, dx$

Put $2e^{2x} + 3 = A(3e^{2x} + 4) + B\frac{d}{dx}(3e^{2x} + 4)$

$\Rightarrow 2e^{2x} + 3 = A(3e^{2x} + 4) + B(6e^{2x})$

Comparing the coefficients, we get

$4A = 3$ and $3A + 6B = 2$

$\therefore$ $A = \frac{3}{4}$ and $B = \frac{-1}{24}$

$\therefore$ $I = \frac{3}{4}\int dx - \frac{1}{24}\int \frac{6e^{2x}}{3e^{2x} + 4}\, dx$

$= \frac{3}{4}x - \frac{1}{24}\log(3e^{2x} + 4) + C$

$\therefore$ $A = \frac{3}{4}, B = -\frac{1}{24}$

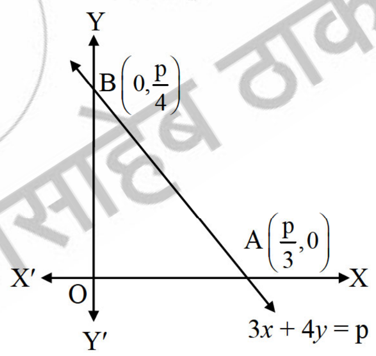
128. (C)

Let the line $3x + 4y = p$ cuts the X and Y axes at points A and B respectively.

$3x + 4y = p$

$\therefore$ $\frac{3x}{p} + \frac{4y}{p} = 1$

$\therefore$ $\frac{x}{\frac{p}{3}} + \frac{y}{\frac{p}{4}} = 1$



This equation is of the form $\frac{x}{a} + \frac{y}{b} = 1$,

where $a = \frac{p}{3}$ and $b = \frac{p}{4}$

$\therefore$ $A \equiv (a, 0) = \left(\frac{p}{3}, 0\right)$ and $B \equiv (0, b) = \left(0, \frac{p}{4}\right)$

$\therefore$ $OA = \frac{p}{3}$ and $OB = \frac{p}{4}$

Given, $A(\Delta OAB) = 24$ sq. units

$\therefore$ $\left|\frac{1}{2} \times OA \times OB\right| = 24$

$\therefore$ $\left|\frac{1}{2} \times \frac{p}{3} \times \frac{p}{4}\right| = 24$

$\therefore$ $p^2 = 576$

$\therefore$ $p = \pm 24$

129. (C)

$f(g(x)) = f\left(\frac{7x + 4}{5x - 3}\right)$

$= \frac{3\left(\frac{7x + 4}{5x - 3}\right) + 4}{5\left(\frac{7x + 4}{5x - 3}\right) - 7}$

$$= \frac{21x+12+20x-12}{35x+20-35x+21}$$

$$= \frac{41x}{41}$$

$$= x$$

Now, $g(f(x)) = g\left(\frac{3x+4}{5x-7}\right)$

$$= \frac{7\left(\frac{3x+4}{5x-7}\right)+4}{5\left(\frac{3x+4}{5x-7}\right)-3}$$

$$= \frac{21x+28+20x-28}{15x+20-15x+21}$$

$$= \frac{41x}{41} = x$$

$\therefore$ $f(g(x)) = g(f(x))$

130. (C)

131. (D)

Volume of parallelopiped $= \left[\bar{a}+2\bar{b} \quad \bar{b}+2\bar{c} \quad \bar{c}+2\bar{a}\right]$

$$= \left(\bar{a}+2\bar{b}\right)\left[\left(\bar{b}+2\bar{c}\right)\times\left(\bar{c}+2\bar{a}\right)\right]$$

$$= \left(\bar{a}+2\bar{b}\right)\left[\left(\bar{b}\times\bar{c}\right)+2\left(\bar{b}\times\bar{a}\right)+2\left(\bar{c}\times\bar{c}\right)+4\left(\bar{c}\times\bar{a}\right)\right]$$

$$= \bar{a}\,.\left(\bar{b}\times\bar{c}\right)+0+0+0+0+8\,\bar{b}\left(\bar{c}\times\bar{a}\right)$$

$$= \left[\bar{a}\ \bar{b}\ \bar{c}\right]+8\left[\bar{b}\ \bar{c}\ \bar{a}\right]$$

$$= 9\left[\bar{a}\ \bar{b}\ \bar{c}\right]$$

$$= 9 \times 4 = 36 \text{ cubic units}$$

Alternate Method:

Volume of parallelopiped $= \left[\bar{a}+2\bar{b} \quad \bar{b}+2\bar{c} \quad \bar{c}+2\bar{a}\right]$

$$= \begin{vmatrix} 1 & 2 & 0 \\ 0 & 1 & 2 \\ 2 & 0 & 1 \end{vmatrix}\left[\bar{a} \quad \bar{b} \quad \bar{c}\right]$$

$$= [1(1)-2(0-4)+0]\left[\bar{a} \quad \bar{b} \quad \bar{c}\right]$$

$$= [1+8]\left[\bar{a} \quad \bar{b} \quad \bar{c}\right]$$

$$= 9 \times 4 \qquad \ldots\left[\because \left[\bar{a} \quad \bar{b} \quad \bar{c}\right]=4\right]$$

$$= 36 \text{ cubic units}$$

132. (C)

Let $\frac{x+3}{5} = \frac{y-1}{2} = \frac{z+4}{3} = \lambda$

$\therefore$ Any general point on this line is

Q$(5\lambda-3, 2\lambda+1, 3\lambda-4)$

Let P $\equiv (0, 2, 3)$

$\therefore$ The direction ratios of PQ are

$5\lambda-3, 2\lambda-1, 3\lambda-7$

Since PQ is perpendicular to given lines

$5(5\lambda - 3) + 2(2\lambda - 1) + 3(3\lambda - 7) = 0$

$\Rightarrow \lambda = 1$

$\therefore$ $Q = (2, 3, -1)$

$\therefore$ $PQ = \sqrt{(0-2)^2 + (2-3)^2 + (3-(-1))^2}$

$= \sqrt{4+1+16} = \sqrt{21}$ units

133. (C)

$(1+e^{-x})(1+y^2)\frac{dy}{dx} = y^2$

$\Rightarrow \left(\frac{1+y^2}{y^2}\right)dy = \left(\frac{1}{1+e^{-x}}\right)dx$

Integrating on both sides, we get

$\int \frac{1}{y^2}dy + \int 1\, dy = \int \frac{e^x}{1+e^x}dx$

$\Rightarrow \frac{-1}{y} + y = \log(1 + e^x) + c$ …(i)

Since the required equation passes through (0, 1).

$c = -\log 2$

$\therefore$ $\frac{-1}{y} + y = \log(1 + e^x) - \log 2$

$\Rightarrow y^2 - 1 = y \log\left(\frac{1+e^x}{2}\right)$

134. (A)

Required number of arrangements

= (Total number of arrangements) – (Number of arrangements in which N's are together)

$= \frac{6!}{2! \times 3!} - \frac{5!}{3!}$

$= 60 - 20 = 40$

135. (B)

$\tan^{-1}\left(\frac{d}{1+a_1a_2}\right) + \tan^{-1}\left(\frac{d}{1+a_2a_3}\right) + \ldots\ldots + \tan^{-1}\left(\frac{d}{1+a_{n-1}a_n}\right)$

$= \tan^{-1}\left(\frac{a_2-a_1}{1+a_1a_2}\right) + \tan^{-1}\left(\frac{a_3-a_2}{1+a_2a_3}\right) + \ldots\ldots + \tan^{-1}\left(\frac{a_n-a_{n-1}}{1+a_{n-1}a_n}\right)$

$= (\tan^{-1} a_2 - \tan^{-1} a_1) + (\tan^{-1} a_3 - \tan^{-1} a_2) + \ldots\ldots + (\tan^{-1} a_n - \tan^{-1} a_{n-1})$

$= \tan^{-1} a_n - \tan^{-1} a_1 = \tan^{-1}\left(\frac{a_n-a_1}{1+a_na_1}\right)$

$= \tan^{-1}\left(\frac{(n-1)d}{1+a_1a_n}\right)$

136. (A)

The equation of plane passing through (1, –1, 2) is $a(x-1) + b(y+1) + c(z-2) = 0$ …(i)

Since plane (i) is perpendicular to the planes $x + 2y - 2z = 4$ and $3x + 2y + z = 6$

$\therefore$ $a + 2b - 2c = 0$ and $3a + 2b + c = 0$

$\Rightarrow \frac{a}{6} = \frac{b}{-7} = \frac{c}{-4}$

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$\therefore$ The equation of the required plane is

$6(x-1)-7(y+1)-4(z-2)=0$

$\Rightarrow 6x-7y-4z-5=0$

137. (B)

$$f(x)=\begin{cases}|x|+[x], & -1\le x<1\\ x+|x|, & 1\le x<2\\ x+[x], & 2\le x\le 3\end{cases}$$

$$\therefore \quad f(x)=\begin{cases}-(x+1), & -1\le x<0\\ x, & 0\le x<1\\ 2x, & 1\le x<2\\ x+2, & 2\le x<3\\ x+3, & x=3\end{cases}$$

$\therefore$ f(x) is discontinuous at x = 0, 1, 3.

138. (B)

$$\frac{\sin A}{a}=\frac{\sin B}{b}=\frac{\sin C}{c}$$

According to the given condition,

In Δ ABC, a = 2b and

A – B = 60° ⇒ A = 60° + B

$$\Rightarrow \frac{\sin(60°+B)}{2b}=\frac{\sin B}{b}$$

$$\Rightarrow \frac{\sin B}{\sin(B+60°)}=\frac{1}{2}$$

$\Rightarrow$ 2 sin B = sin B cos 60° + cos B sin 60°

$$\Rightarrow \frac{3}{2}\sin B=\frac{\sqrt{3}}{2}\cos B$$

$\therefore$ $\tan B=\frac{1}{\sqrt{3}} \Rightarrow B=30°$

$\therefore$ A = 30° + 60° = 90°

$\therefore$ ΔABC is right angled.

139. (A)

$y=e^{x^2}$ ....(i)

$y=e^{x^2}\sin x$ ....(ii)

From (i) and (ii), we get

$e^{x^2}=e^{x^2}\sin x$

$\therefore$ $\sin x=1 \Rightarrow x=\frac{\pi}{2}$

Slope of tangent to (i) at $x=\frac{\pi}{2}$ is given by $\left(\frac{dy}{dx}\right)_{x=\pi/2}=\left[2xe^{x^2}\right]_{x=\frac{\pi}{2}}=\pi e^{\frac{\pi^2}{4}}$

Slope of tangent to (ii) at $x=\frac{\pi}{2}$ is given by

$$\left(\frac{dy}{dx}\right)_{x=\pi/2}=\left[2xe^{x^2}\sin x+e^{x^2}\cos x\right]_{x=\frac{\pi}{2}}=\pi e^{\frac{\pi^2}{4}}$$

Since both tangents have equal slopes, the angle between them is zero.

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140. (C)

$\left(1+y_1^2\right)^{2/3} = y_2$

$\Rightarrow \left(1+y_1^2\right)^2 = \left(y_2\right)^3$

∴ order(n) = 2, degree(m) = 3

∴ $\frac{m+n}{m-n} = \frac{3+2}{3-2} = 5$

141. (C)

q = Probability that the candidate can solve any problem = $\frac{4}{5}$

$p = 1 - \frac{4}{5} = \frac{1}{5}$

Also, n = 50

∴ Required probability = P(X < 2)

= P(X = 0) + P(X = 1)

$= {}^{50}C_0\left(\frac{1}{5}\right)^0\left(\frac{4}{5}\right)^{50} + {}^{50}C_1\left(\frac{1}{5}\right)^1\left(\frac{4}{5}\right)^{49} = \left(\frac{4}{5}\right)^{50} + 50\left(\frac{1}{5}\right)\left(\frac{4}{5}\right)^{49} = \left(\frac{4}{5}+\frac{50}{5}\right)\left(\frac{4}{5}\right)^{49} = \left(\frac{54}{5}\right)\left(\frac{4}{5}\right)^{49}$

142. (C)

$\sqrt{3}$ cosec θ + 2 = 0

∴ cosec $\theta = -\frac{2}{\sqrt{3}}$

∴ cosec θ = – cosec$\left(\frac{\pi}{3}\right)$

∴ cosec θ = cosec$\left(\pi+\frac{\pi}{3}\right)$ = cosec$\left(\frac{4\pi}{3}\right)$ and cosec θ = cosec$\left(2\pi-\frac{\pi}{3}\right)$ = cosec$\left(\frac{5\pi}{3}\right)$

such that $0 < \frac{4\pi}{3} < 2\pi$ and $0 < \frac{5\pi}{3} < 2\pi$

∴ The required principal solutions are $\theta = \frac{4\pi}{3}$ and $\theta = \frac{5\pi}{3}$.

143. (C)

cos 2θ = sin θ ⇒ 1 – 2 sin²θ = sin θ

⇒ 2 sin²θ + sin θ – 1 = 0

⇒ (2 sin θ – 1) (sin θ + 1) = 0

⇒ sin θ = $\frac{1}{2}$ or sin θ = –1

∴ $\sin\theta = \frac{1}{2} = \sin\frac{\pi}{6} \Rightarrow \theta = n\pi + (-1)^n\frac{\pi}{6}$

and $\sin\theta = -1 = \sin\frac{3\pi}{2}$

$\Rightarrow \theta = m\pi + (-1)^m\frac{3\pi}{2}$

∴ $\theta = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{3\pi}{2}$

∴ number of solutions = 3

144. (C)

Required line is perpendicular to the lines $\frac{x-2}{3}=\frac{y-1}{2}=\frac{z+1}{-2}$ and $\frac{x}{2}=\frac{y}{-3}=\frac{z}{1}$

$\therefore$ Required line is parallel to vector $\bar{b}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 3 & 2 & -2\\ 2 & -3 & 1\end{vmatrix}=-4\hat{i}-7\hat{j}-13\hat{k}$

$\therefore$ The equation of the required line is $\bar{r}=\left(\hat{i}+2\hat{j}+3\hat{k}\right)+\lambda\left(-4\hat{i}-7\hat{j}-13\hat{k}\right)$

145. (C)

Let $y=\tan^{-1}\left(\sqrt{\frac{\sqrt{1+x^2}+x}{\sqrt{1+x^2}-x}}\right)$

Put $x=\tan\theta$, then $\theta=\tan^{-1}x$

$\therefore$ $y=\tan^{-1}\left(\sqrt{\frac{\sqrt{1+\tan^2\theta}+\tan\theta}{\sqrt{1+\tan^2\theta}-\tan\theta}}\right)$

$=\tan^{-1}\left(\sqrt{\frac{\sqrt{\sec^2\theta}+\tan\theta}{\sqrt{\sec^2\theta}-\tan\theta}}\right)$

$=\tan^{-1}\left(\sqrt{\frac{\sec\theta+\tan\theta}{\sec\theta-\tan\theta}}\right)$

$=\tan^{-1}\left(\sqrt{\frac{\sec\theta+\tan\theta}{\sec\theta-\tan\theta}\times\frac{\sec\theta+\tan\theta}{\sec\theta+\tan\theta}}\right)$

$=\tan^{-1}\sqrt{\frac{(\sec\theta+\tan\theta)^2}{\sec^2\theta-\tan^2\theta}}$

$=\tan^{-1}(\sec\theta+\tan\theta)$

$=\tan^{-1}\left(\frac{1+\sin\theta}{\cos\theta}\right)$

$=\tan^{-1}\frac{\left(\sin\left(\frac{\theta}{2}\right)+\cos\left(\frac{\theta}{2}\right)\right)^2}{\left(\cos^2\left(\frac{\theta}{2}\right)-\sin^2\left(\frac{\theta}{2}\right)\right)}$

$=\tan^{-1}\left(\frac{\cos\left(\frac{\theta}{2}\right)+\sin\left(\frac{\theta}{2}\right)}{\cos\left(\frac{\theta}{2}\right)-\sin\left(\frac{\theta}{2}\right)}\right)$

$=\tan^{-1}\left(\frac{1+\tan\left(\frac{\theta}{2}\right)}{1-\tan\left(\frac{\theta}{2}\right)}\right)$

$=\tan^{-1}\left(\frac{\tan\left(\frac{\pi}{4}\right)+\tan\left(\frac{\theta}{2}\right)}{1-\tan\left(\frac{\pi}{4}\right).\tan\left(\frac{\theta}{2}\right)}\right)=\tan^{-1}\left(\tan\left(\frac{\pi}{4}+\frac{\theta}{2}\right)\right)=\frac{\pi}{4}+\frac{0}{2}=\frac{\pi}{4}+\frac{1}{2}\tan^{-1}x$

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Differentiating w. r. t. x, we get

$$\frac{dy}{dx} = \frac{d}{dx}\left(\frac{\pi}{4}+\frac{1}{2}\tan^{-1}x\right) = \frac{1}{2}\left(\frac{1}{1+x^2}\right).$$

146. (D)

Let $y = \log_e 2 \frac{d}{dx}(\log_{\cos x} \text{cosec}\, x)$

$$= \log_e 2 \frac{d}{dx}\left(\frac{\log_e \text{cosec}\, x}{\log_e \cos x}\right)$$

$$= \log_e 2 \frac{\left(\log_e \cos x \cdot \frac{d}{dx}\log_e \text{cosec}x - \log_e \text{cosec}x \cdot \frac{d}{dx}\log_e \cos x\right)}{(\log_e \cos x)^2}$$

$$= \log_e 2 \left[\frac{\log_e \cos x \cdot \frac{1}{\text{cosec}x} \times -\text{cosec}x \cdot \cot x - \log_e \text{cosec}x \times \frac{1}{\cos x} \times -\sin x}{(\log_e \cos x)^2}\right]$$

$$= \log_e 2 \left[\frac{-\log_e \cos x \cdot \cot x + \log_e \text{cosec}x \cdot \tan x}{(\log_e \cos x)^2}\right]$$

$$\therefore \quad y\Big|_{\text{at } x=\frac{\pi}{4}} = \log_e 2 \left[\frac{-\log_e \cos\frac{\pi}{4} \cdot \cot\frac{\pi}{4} + \log_e \text{cosec}\frac{\pi}{4} \cdot \tan\frac{\pi}{4}}{\left(\log_e \cos\frac{\pi}{4}\right)^2}\right]$$

$$= \log_e 2 \left[\frac{-\log_e \frac{1}{\sqrt{2}} \times 1 + \log_e \sqrt{2}}{\left(\log_e \frac{1}{\sqrt{2}}\right)^2}\right]$$

$$= \log_e 2 \left[\frac{\log_e 2}{\frac{1}{4}(\log_e 2)^2}\right]$$

$$= 4$$

147. (B)

$(\bar{a}+\bar{b})\cdot[(\bar{b}+\bar{c})\times(\bar{a}+\bar{b}+\bar{c})]$

$= (\bar{a}+\bar{b})\cdot\left[\bar{b}\times\bar{a}+\bar{b}\times\bar{c}+\bar{c}\times\bar{a}+\bar{c}\times\bar{b}\right]$

$= \left[\bar{a}\,\bar{b}\,\bar{a}\right]+\left[\bar{a}\,\bar{b}\,\bar{c}\right]+\left[\bar{a}\,\bar{c}\,\bar{a}\right]+\left[\bar{a}\,\bar{c}\,\bar{b}\right]+\left[\bar{b}\,\bar{b}\,\bar{a}\right]+\left[\bar{b}\,\bar{b}\,\bar{c}\right]+\left[\bar{b}\,\bar{c}\,\bar{a}\right]+\left[\bar{b}\,\bar{c}\,\bar{b}\right]$

$= 0 + \left[\bar{a}\,\bar{b}\,\bar{c}\right] + 0 + \left[\bar{a}\,\bar{c}\,\bar{b}\right] + 0 + 0 + \left[\bar{b}\,\bar{c}\,\bar{a}\right] + 0$

$= \left[\bar{a}\,\bar{b}\,\bar{c}\right] - \left[\bar{a}\,\bar{b}\,\bar{c}\right] + \left[\bar{a}\,\bar{b}\,\bar{c}\right]$

$= \left[\bar{a}\,\bar{b}\,\bar{c}\right]$

148. (C)

Line is perpendicular to normal of plane

$\Rightarrow \left(2\hat{i}-\hat{j}+3\hat{k}\right).\left(l\hat{i}+m\hat{j}-\hat{k}\right) = 0$

$2l - m - 3 = 0$ …(i)

$(3, -2, -4)$ lies on the plane $lx + my - z = 9$

∴ $3l - 2m + 4 = 9$

$\Rightarrow 3l - 2m = 5$ …(ii)

Solving (i) and (ii)

$l = 1, m = -1$

$l^2 + m^2 = 2$

149. (D)

$f(x) = \int \frac{2x}{(x^2+1)(x^2+3)} dx$

Put $x^2 = t$

$\Rightarrow 2x\, dx = dt$

∴ $f(x) = \int \frac{dt}{(t+1)(t+3)}$

$= \frac{1}{2}\int \frac{2}{(t+1)(t+3)} dt$

$= \frac{1}{2}\int \frac{(t+3)-(t+1)}{(t+1)(t+3)} dt$

$= \frac{1}{2}\int \left(\frac{1}{t+1} - \frac{1}{t+3}\right) dt$

$= \frac{1}{2}\left(\log_e|t+1| - \log_e|t+3|\right) + c = \frac{1}{2}\log_e\left|\frac{t+1}{t+3}\right| + c$

∴ $f(x) = \frac{1}{2}\log_e\left|\frac{x^2+1}{x^2+3}\right| + c$

∴ $f(3) = \frac{1}{2}\log_e\left(\frac{10}{12}\right) + c$

∴ $f(3) = \frac{1}{2}(\log_e 5 - \log_e 6) + c$

But, $f(3) = \frac{1}{2}(\log_e 5 - \log_e 6)$ ...[Given]

∴ $c = 0$

∴ $f(4) = \frac{1}{2}\log_e\left(\frac{17}{19}\right) = \frac{1}{2}(\log_e 17 - \log_e 19)$

150. (A)

Let x and y be two natural numbers such that $x + y = 10$ and the product is xy.

$xy = x(10 - x) = 10x - x^2 = f(x)$

∴ $f'(x) = 10 - 2x$

∴ $f''(x) = -2$

Roots of $f'(x) = 0$,

i.e., $10 - 2x = 0$, i.e., $x = 5$

$f'(5) = 10 - 10 = 0$

∴ f is maximum when $x = 5, y = 5$

∴ The product is maximum if $x = 5, y = 5$